The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 5
Name of the Examination	: Final Examination
Course Code - Title	: ADU5304-Operational Research
Academic Year	: 2023/24
Date	: 03/04/2024
Time	: 9.30 a.m – 11.30 a.m
Duration	: 2hrs.

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 6 questions in 6 pages.
3. Answer any 4 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered an exam offense will lead to punishment.
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script

Question 01

(a) State four characteristics that a competitive situation should have if it is to be called a competitive game.

(b) Discuss minimax and maximin principles.

(c) Explain two-person zero-sum game.

(d) Solve the two-person zero-sum game with the following pay-off matrix.

Player B strategies

		I	II	III	IV	V
Player A strategies	1	7	5	2	3	9
	2	10	8	7	4	5
	3	9	12	0	2	1
	4	11	-2	-1	3	4

Question 02

(a) Consider the following payoff matrix for 2×2 two-person zero-sum game which does not have any saddle point:

Player B

Player A	B_1	B_2
A_1	a_{11}	a_{12}
A_2	a_{21}	a_{22}

Write down the formulas for optimum mixed strategies of Player *A* and Player *B* and the value of the game.

(b) Consider a modified form of "matching biased coins" game problem. The matching player is paid Rs.8.00 if the coins turn both heads and Rs.1.00 if both turn tail. The nonmatching player is paid Rs.3.00 when the two coins do not match. Given the choice of being the matching or non-matching player, which one would you choose and what would be your strategy?

Question 03

(a) Briefly explain the following terms in queuing theory:

(i) Queue discipline.

(ii) Service mechanism.

(iii)The Capacity of the system

(b) Students arrive in a Poisson distribution at the school medical Centre for getting counselling service at the rate of one per hour. Currently, only one student can be handled at a time. Students spend on average 30 minutes receiving doctor's care. The service time is found to have an exponential distribution.

(i) What is the probability that a student arriving at the Centre will have to wait?

(ii) Find the average length of the queue that forms.

(iii)Find the average time a student spends in the system.

(iv)What is the probability that there will be five or more students waiting for the service?

(v) Determine the fraction of the time that there are no students.

Question 04

The mean rate of arrival of planes at an airport during the peak period is 30 per hour, but the actual number of arrivals is any hour follows a Poisson distribution. The airport can land 60 planes per hour on an average in good weather, or 40 per hour in bad weather, but the actual number landed in any hour follows a Poisson distribution with a respective average. When there is congestion, the planes are forced to fly over the field in the stack awaiting the landing of other planes that arrived earlier.

a) How many planes would be flying over the stack on an average in good weather and in bad weather?

b) How long a plane would be in the stack and the process of landing in good weather and bad weather?

c) How much stack landing time to allow so that priority to land out of order would have to be requested only once in twenty.

Question 05

(a) Define the term "inventory".

(b) Write down the advantages and disadvantages of having inventories.

(c) The John Equipment company estimates it carrying cost at 15% and its ordering cost at $9 per order. The estimated annual requirement is 48,000 units at a price of $4 per unit.

a) What is the most economical number of units to order.

b) How many orders should be placed in a year.

c) How often should an order be placed?

(Assume that the stock replenishment is instantaneous, the demand is uniform and no shortages allowed.)

Question 06

(a) Briefly explain the following terms used in Inventory Management:

(i) Carrying cost

(ii) Shortage cost

(iii)Ordering cost

(b) The demand for an item is 18,000 units/year. The demand for an item is units/year. The cost of one purchase is Rs 400.00. The holding cost is Rs 1.20 per unit per year. The shortage cost is Rs 5.00 per unit per year. Determine:

(i) The optimum order quantity.
(ii) The time between orders.
(iii)The number of orders per year.
(iv)The optimum shortages.
(v) The maximum inventory.
(vi)The time of items being held.

(Assume that the stock replenishment is instantaneous, and the demand is uniform.)

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Formulas (in the usual notation)

(M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

$$\text{P(queuesize} \geq \text{n)} = \rho^n$$

$$E(n) = \frac{\lambda}{\mu-\lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad E(v) = \frac{1}{\mu-\lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$$

(M/M/1): (N/FIFO) Queueing System

$$P_n \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}\ ; & \rho \neq 1 \\ \dfrac{1}{N+1}; & \rho = 1 \end{cases}$$

$$E(m) = \frac{\rho^2[1 - N\rho^{N-1} + (N-1)\rho^N]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(n) = \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(w) = E(v) - \frac{1}{\mu} \ or\ E(w) = \frac{E(m)}{\lambda'}$$

$$E(v) = \frac{E(n)}{\lambda'}, \qquad where\ \lambda' = \lambda(1-P_N)$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ; 1 \leq n \leq C \\ \dfrac{1}{C^{n-c}C!}\rho^n P_0 & ;\ n > C \end{cases}$$

$$E(m) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^c P_0}{(C-1)!\,(C\mu-\lambda)^2}$$

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1}$$

$$E(n) = E(m) + \frac{\lambda}{\mu} \qquad E(w) = \frac{1}{\lambda}E(m)$$

$$E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; 0 \le n \le C \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;\ n > C \end{cases}$$

$$E(w) = E(v) - \frac{1}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=C}^{\infty} \frac{1}{C^{n-C}C!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!}$$

$$E(v) = \frac{E(n)}{\lambda'}\ , \quad where\ \lambda' \lambda(1 - P_N)$$

(M/M/R):(K/GD) Model

$$P_n \begin{cases} \binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; 0 \le n \le R \\ \binom{K}{n} \frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;\ R \le n \le K \end{cases}$$

$$E(m) = \sum_{n=R}^{K} (n - R)P_n$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(v) = \frac{E(n)}{\lambda[K-E(n)}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(w) = \frac{E(m)}{\lambda[K - E(n)]}$$