The Open University of Sri Lanka
Faculty of Natural Sciences
B. Sc./ B. Ed. Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title Code	**: Newtonian Mechanics II**
Course Title Code	**: ADU5303**
Academic Year	**: 2023/22024**
Date	**: 01.04.2024**
Time	**: 1.30 p.m. To 3.30 p.m.**
Duration	**: Two Hours.**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(4)** pages.
3. Answer any **Four (4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary.
6. Involvement in any activity that is considered as an exam offense will lead to punishment.
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script.

1. (a) In the usual notation, show that in Spherical polar coordinates, the velocity $\underline{v}$ and acceleration $\underline{a}$ of a particle are given by

$$\underline{v} = \dot{r}\underline{e_r} + r\dot{\theta}\underline{e_\theta} + r\dot{\phi}\sin\theta\,\underline{e_\phi} \quad \text{and}$$

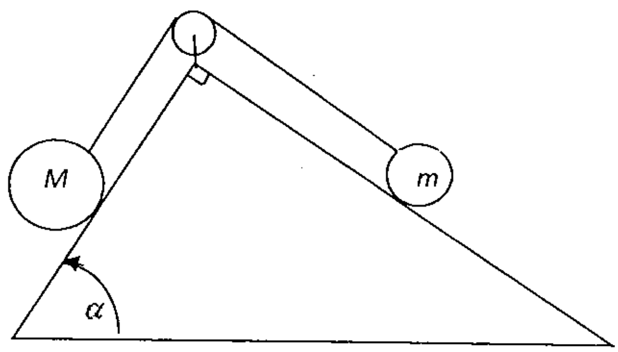
$$\underline{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\underline{e_r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\dot{\phi}^2\sin\theta\cos\theta\right)\underline{e_\theta} + \left(\frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\,\dot{\phi}\right)\right)\underline{e_\phi}.$$

(b) A particle of mass m is projected on the inner surface of a hollow sphere of radius a, with horizontal velocity u from a position $\theta = \alpha$ from the downward vertical diameter.

Show that $a^2\dot{\theta}^2 + u^2\left(\frac{\sin^2\alpha}{\sin^2\theta} - 1\right) = 2ag(\cos\theta - \cos\alpha)$.

Let R be the reaction on the particle then show that
$R = mg(3\cos\theta - 2\cos\alpha) + m\frac{u^2}{a}$.

2. Two particles of mass M and m are connected by an inelastic string and placed on two fixed inclined planes as shown in the diagram. The coefficient of friction between the particles and the planes is μ. Using D'Alembert's principle, determine the acceleration of each particle and tension in the string.

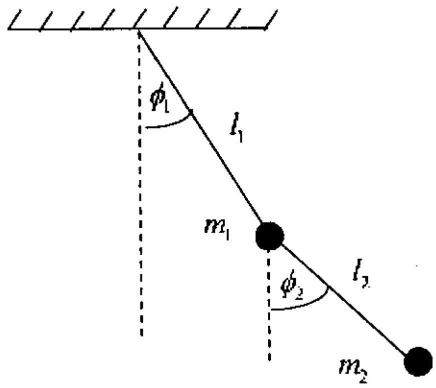


3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) An object is projected vertically upward from a point on the surface of the earth with latitude λ with speed v_0. Find the position of the particle at time t.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \; j = 1, 2 \ldots, n.$

(b) The double pendulum swinging in a vertical plane consists of two bobs of masses m_1 and m_2 at ends of two weightless rods of lengths l_1 and l_2 and one of them is fixed to a rigid support as shown in figure.



(i) Show that the Lagrangian of the system is given by

$$L = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\phi}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\phi}_2^2 + m_2 l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) + (m_1 + m_2) g l_1 \cos\phi_1 + m_2 g l_2 \cos\phi_2 .$$

(ii) Hence, obtain the Lagrange's equations of motion.

5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) If a rectangular parallelepiped with its edges $2a, 2a, 2b$ rotates about its center of gravity under no forces. Prove that, its angular velocity about one principal axis is constant and about the other axis it is periodic.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(b) The Hamiltonian of a dynamical system is given by $H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$

where a, b are constants. Obtain Hamilton's equations of motion and hence find p_1, q_1, p_2 and q_2 at time t.