THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF SCIENCE DEGREE PROGRAMME – LEVEL 05

FINAL EXAMINATION - 2023/2024

PHU 5313 - ADVANCED ELECTROMAGNETISM

Duration: TWO (02) HOURS

Date 27th March 2024 **Time 1.30 pm – 3.30 pm**

$C = 2.99\times10^{8}\ ms^{-1}$, $\varepsilon_0 = 8.85\times 10^{-12}\ Fm^{-1}$, $\mu_0 = 4\pi\times 10^{-7}\ Hm^{-1}$

Answer any Four (04) questions only

1. (a) What would be the resultant when del (∇) operator acts on a scalar field? Give an example for this kind of operation that you find in electromagnetism for a scalar field.

   (b) What are the two ways the ∇ can operate on a vector field? Write down the expressions for operation of ∇ on electric field and magnetic field at static condition in the above two ways.

   (c) How do you write down the following expressions mathematically by taking A and f to be a vector field and a scalar field respectively?

   (i) Gradient of divergence of a vector field, (ii) Divergence of gradient of a scalar field

   (iii) Divergence of curl of a vector field, (iv) Curl of gradient of a scalar field

   (v) Curl of curl of a vector field

   (d) Identify the expressions that produce a vector field from among the above five operations.

   (e) Prove that for a given vector field, A and a scalar field, f

   (i) $\nabla.(\nabla f) = \nabla^2 f$

   (ii) $\nabla.(\nabla \times \mathbf{A}) = 0$

2. (a) State the divergence theorem and the Stokes theorem.

   (b) A cube of 2 m of a side is placed in an electric flux density, $\mathbf{D} = (10x^3/3)\mathbf{a}_x\ Cm^{-2}$ in such a way that its three faces align with the cartesian coordinates. Verify the divergence theorem.

(c) Show that the curl of the electric field given in (b) acts on any face of the cube is zero with the help of the Stokes theorem.

(d) Using the divergence theorem and the Stokes theorem convert the following equations to their differential form.

(i) $\oint_S \boldsymbol{E} . d\boldsymbol{s} = \frac{1}{\varepsilon_0} \int_V \rho \, dv$

(ii) $\oint_S \boldsymbol{B} . d\boldsymbol{s} = 0$

(iii) $\oint_C \boldsymbol{E} . d\boldsymbol{l} = - \int_S \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{S}$

(iv) $\oint_C \boldsymbol{B} . d\boldsymbol{l} = \mu_0 \int_S (\boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}) d\boldsymbol{S}$

(e) State the Helmholtz theorem and explain its significance.

Show that $\boldsymbol{r} = x^2 \hat{\boldsymbol{r}}_x - 2xy \hat{\boldsymbol{r}}_y$ represents a magnetic field and find its vector potential.

3. (a) State the Coulomb's law. Hence show that the force act on a charge Q due to the presence of charge distribution $q_1, q_2, \ldots q_n$ at points $r_1, r_2, \ldots r_n$ respectively is given by

$$\boldsymbol{F} = \frac{Q}{4\pi\varepsilon} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{\boldsymbol{r}}_i$$

Where the symbols have their usual meaning.

(b) How do you modify this relation for a continuous charge distribution? Derive an expression for electric field strength of continuous charge distribution by considering the relation between electric force and field.

(c) Find the electric field at distance z above the centre of a flat circular disc of radius R, which carries uniform surface charge σ per unit area.

(d) Briefly explain how the results of ice pail experiment inferred to develop the Gauss's Law

(e) Using Gauss's law show that the electric field of an infinite plane of surface with charge density σ is independent of the distance r.

4. (a) A charge Q is placed at a distance r away from the centre of a hollow conducting sphere of radius R (R < r) which is earthed. State two ways to find the charge induced on the conducting sphere.

(b) Derive an expression for the induced charge on the sphere by using any one of the methods you described above.

(c) To obtain an identical electric field as in the system described in (a), where do you place the total charge induced on the conducting sphere from the centre without grounding it?

(d) Show that the boundary condition on the hollow conducting sphere is same in both the situations described in (a) and (c).

(e) 5 μC charge is placed on the sphere in the system described in (a) after disconnecting it from earth. Calculate the radius of the sphere if the potential on the sphere is 10^3 V.

5. (a) State the situation where you have to use the Laplace equation instead of Gauss's theorem to find electrostatic fields in problems.

(b) Write down the Laplace equation in a rectangular coordinate system for an electrostatic potential V.

(c) What will guarantee that the solution of variable separable form produces the solution for the Laplace equation? Write down the potential in variable separable form.

(d) Substituting the potential in variable separable form show that the,

$\frac{1}{X}\frac{d^2X}{dx^2} = c_1$, $\frac{1}{Y}\frac{d^2Y}{dy^2} = c_2$, and $\frac{1}{Z}\frac{d^2Z}{dz^2} = c_3$ and also $C_1 + C_2 + C_3 = 0$, where C_1, C_2 and C_3 are constants.

(e) Two grounded infinite metal plates lie parallel to the xz plane that extend from x = 0 to infinity, where one is at y = 0 and the other is at y = a. The end at x = 0 is closed with an insulated infinitely long metal strip at potential v_0.

(i) Write down the boundary conditions of this problem.

(ii) Find the potential at any point between the two plates by solving the Laplace equation.

6. (a) Write down the laws in electromagnetism that relate to each of Maxwell's equations for static electromagnetic fields.

(b) State the related laws in electromagnetism that used to develop Maxwell's time varying equations.

(c) Derive Maxwell's equations for free space from equations of time varying electromagnetic fields.

(d) Using Maxwell's equations in free space show that the electric and the magnetic fields can be transmitted through free space with velocity $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ where symbols have their usual meaning.

(e) An electromagnetic wave of 100 GHz propagating in free space has an amplitude of 10 V/m.

(i) Calculate the propagation velocity and the wavelength of the electromagnetic wave.

(ii) What would be the propagation velocity and wavelength when it is in a dielectric medium of relative permittivity of 4 and relative permeability of 9.

*****