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The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2522 - Combined Mathematics 4 – Level 2

Final Examination 2024/2025

Date: 15-03-2025 From 9:30 am. To 12:30 pm.

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. Two uniform rods AB and BC of equal length but of weights W and $3W$ respectively are freely joined together at B. The rods stand in a vertical plane with the ends A and C on smooth horizontal ground. The rods are kept inclined at 60^0 horizontally to each other by joining the midpoints of the rods by a light rod. Find the reactions at points A and C.

2. A uniform ladder rests against a smooth vertical wall with one end on rough horizontal ground. The weight of the ladder is $10\ W$. The ladder is just about to slip when inclined at 30° to the **vertical**. Calculate the coefficient of friction between the ladder and the ground.

3. A gun of mass M kg fires a shell of mass $m\ kg$ and recoils. The shell exits the barrel with speed u ms^{-1}. Find the velocity with which the barrel begins to recoil in each of the following cases.
   (a) If the barrel is horizontal.
   (b) If the barrel is inclined at an angle α to the horizontal.

4. A particle A of mass $m\ kg$ and a particle B of mass $2m\ kg$ move along a straight line towards each other on a smooth horizontal table with speed $4u\ ms^{-1}$ and $u\ ms^{-1}$ respectively, and collide directly. The coefficient of restitution between A and B is 0.8. Show that after the collision the particles A and B move away from each other along the same straight line.

5. Show that the centre of gravity of a uniform solid right circular cone of height h and radius r is at a distance $\frac{h}{4}$ from the center of the base of the cone.

6. A particle of mass $m\ kg$ is attached to end B of a light inelastic string AB of length $5a$. The other end A of the string is fixed. The particle moves with constant angular speed in a horizontal circle of radius $3a$ with the centre O vertically below A. Calculate the tension in the string and the angular speed of the particle.

7. A small smooth particle P of mass m *kg* is free to move under gravity in a thin smooth circular tube of radius r and centre O fixed in a vertical plane. The particle is projected horizontally from the lowest point of the tube with speed $\sqrt{3gr}$. If v is the speed of the particle when OP makes an angle θ $(0 < \theta < 90^o)$ with the downward vertical. Show that $v^2 = gr(1+2\cos\theta)$

8. A particle performs linear simple harmonic motion of amplitude *3 m*. When the particle is at a distance of *2 m* from the center of the path of motion, its speed is *2 ms*$^{-1}$. Find the periodic time of the motion.

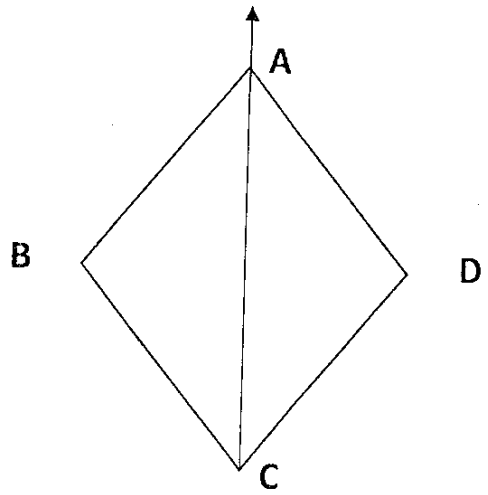
9. Let A and B be two events of a sample space E. In the usual notation, it is given that $P(A) = \frac{3}{5}$, $P(B/A) = \frac{1}{4}$ and $P(A \cup B) = \frac{4}{5}$. Find $P(A)$ and $P(A/B)$

10. The ***mode*** and ***mean*** of the ten observations *8, 6, 9, 12, 4, 6, 8, 10, x, y* are *6* and *8* respectively where $x,\ y \in \mathbb{N}$. Find x and y and the median of the above ten observations.

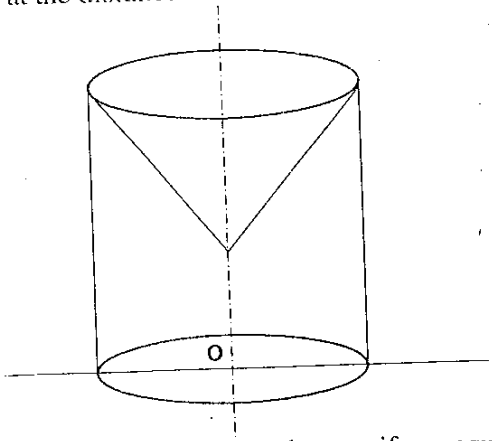
PART B

11. A uniform ladder of mass *30 kg* and length *10 m* rests against a smooth vertical wall with its lower end on rough ground. The coefficient of friction between the ground and the ladder is *0.3*. The ladder is inclined at an angle θ to the horizontal where $\tan\theta = 2$. Find how far a boy of mass *30 kg* can ascend the ladder without slipping.

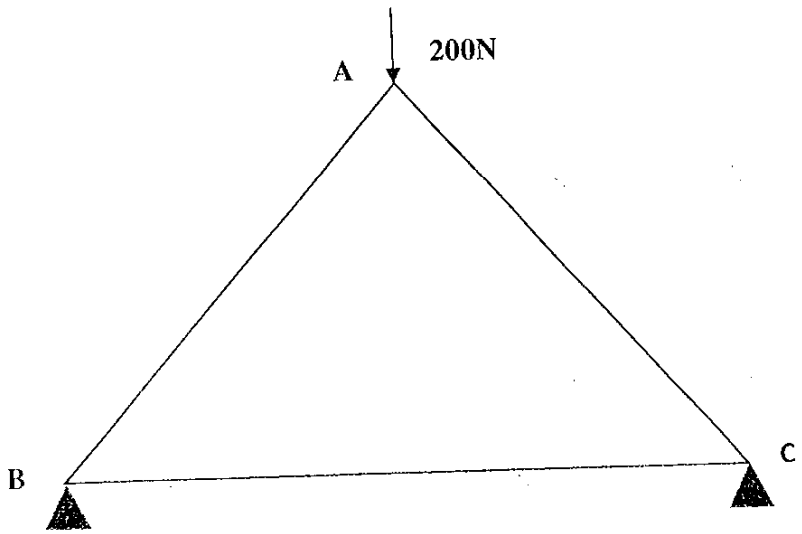
12. Four uniform rods *AB, BC, CD, DA* each of length l and weight $2W$ are freely jointed at their ends. A light rod is attached at A and C to form a framework (See the figure). The framework is freely suspended from A and is maintained at equilibrium. The angle BÂD is 2θ. Find the force exerted by the light rod AC and the reaction at the joint B.



13. From a uniform solid right circular cylinder of radius r and height $2h$, a solid right circular cone of radius r and height h is bored out so that the base of the cone coincides with one end of the cylinder. Find the centre of gravity of the remainder is on the axis of symmetry of the cylinder at the distance from the centre of the base of the cylinder **O** (See the figure).



14. **ABC** is a frame obtained by joining three uniform equal light rods **AB, BC** and **AC. B** and **C** rest on two pegs at the same horizontal level and **A** carries a load of ***200N***. (See the figure) Find the reaction at **B** and **C**. Draw a stress diagram by using **Bow's notation**. Hence find the stress in each rod, distinguishing between tension and thrust.



15. One end of a light elastic string of natural length l and modulus of elasticity $2mg$ is tied to a fixed point O and a particle P of mass m kg is attached to the other end. The particle is released from rest at O. Find the velocity of the particle P when it passes through the point A, where $OA = l$. **It is given** that the length of the string x $(\geq l)$ satisfies the equation

$\ddot{x} + \frac{2g}{l}\left(x - \frac{3l}{2}\right) = 0$. Taking $X = x - \frac{3l}{2}$, express the above equation in the form

$\ddot{X} = -\omega^2 X$ where ω (>0) is a constant to be determined. Assume that

$\dot{X}^2 = \omega^2(c^2 - X^2)$. Find the amplitude c of this simple harmonic motion. Let L be the lowest point reached by the particle P. Find the time taken by P to move from A to L.

16. A particle P of mass m kg is attached to one end A of a light inelastic string of length l, the other end O being fixed. Initially the particle P hangs vertically below O. It is projected horizontally with speed u ms^{-1} from lowest point.

    (a) Find v^2, If v is the speed of the particle when OA makes an angle θ ($0< \theta <90^o$) with the downward vertical. Find the tension of the String.

    (b) When the particle leaves its circular path, Find the OA makes an angle with the upward vertical.

17. (a) The probabilities of producing nails by machines X, Y and Z are $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively. It is known that the 1%, 3% and 5% of the nails produced from X, Y and Z are defective.

    (i) What is the probability that a nail produced is found to be defective.
    (ii) If a nail is found to be defective what is the probability that this nail is produced by machine X.

(b). The following table gives the distribution of marks obtained by a group of 100 students for their answers for Combined Mathematics modal paper.

Marks Range	Number of students
0 – 20	5
20 – 40	30
40 – 60	50
60 – 80	10
80 – 100	5

Find the ***mean*** and ***standard deviation*** of this distribution.