The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2520 - Combined Mathematics 2 – Level 2

Final Examination 2024/2025

Date: 09-03-2025 From 1:30 pm. To 4:30 pm.

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. If $\underline{a}, \underline{b}$ are two vectors such that $|\underline{a}| = |\underline{b}| = |\underline{a} + \underline{b}|$, Find the angle between $\underline{a}$ and $\underline{b}$.

2. If the vectors $\underline{c} = \lambda \underline{i} - 4(\lambda + 8)\underline{j}$ and $\underline{d} = \lambda \underline{i} + \underline{j}$ are perpendicular to each other, find the possible values for λ ($\lambda \in \mathbb{R}$).

3. A particle of weight 10 N is suspended from points A and B of 100 cm apart and in the same horizontal line, by two strings of length 80 cm and 60 cm. Find the tension in each string.

4. $ABCDEF$ is a regular hexagon. AB is the horizontal base. Forces $2\sqrt{3}$, 6, $5\sqrt{3}$, 8, and $\sqrt{3}$ Newton act along AC, AB, EC, FC and FD respectively in the directions indicated by the order of the letters. Resolving vertically and resolving horizontally find the resultant force and its direction.

5. $ABCDEF$ is a regular hexagon. Prove that $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD}$.

6. A stone is catapulted vertically upward from the ground level with a velocity of 24.5 ms^{-1}. Modelling the stone as a particle moving under gravity alone, find for how long its height exceeds 29.4 m. The acceleration due to gravity is 9.8 ms^{-2}.

7. The Wind blows from the South - East at $30kmh^{-1}$. While a man is going to the West, the wind appears to be blowing from the South. Find the magnitude of the velocity of the man.

8. Two particles P and Q of masses m kg and M kg $(M > m)$ respectively are connected by a light inextensible string. Particle P rests on a smooth horizontal table. The string passes over a smooth pulley fixed at the edge of the table and Q hangs vertically. The system is released from rest. Find the following in terms of gravitational acceleration g .
   (a) the acceleration of Q.
   (b) the tension of the string.

9. A particle is moving along a straight line and passes points, A, B, and C on the line with constant acceleration at $t = 0$, $t = 2s$, $t = 6s$ respectively. If AC equals $80m$ and the velocity of the particle at A is $6ms^{-1}$ then find the acceleration of the particle and the distance AB.

10. A lorry of mass M kg and constant power H W moves downwards along a straight road of inclination α to the horizontal. There is a constant resistance of $R\ (> Mgsin\alpha)N$ to its motion. At a certain instant, the acceleration of the lorry is f ms^{-2}. Find the velocity of the lorry at this instant.

PART B

11. In a motor car race, motor car A is about to finish the race and it has to go only $1100m$ distance. At that moment the velocity is $38.5ms^{-1}$ and the constant acceleration $0.44ms^{-2}$. In this instance, motor car B is 220m behind motor car A and at that time the velocity of B is 48.4 ms^{-1} and the constant acceleration is 0.55 ms^{-2}

    (a) Sketch the velocity-time graph.
    (b) Show that, before the finishing line, motor car B overtakes motor car A at a distance $242m$.

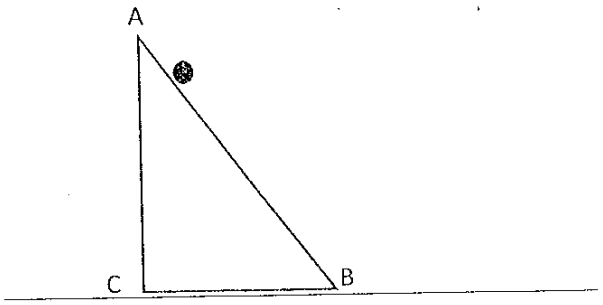
12. Consider a particle which is projected from a point O level ground with a velocity u at an angle α to the horizontal. Take g as the acceleration due to gravity. The particle just clears two vertical walls of height λH ($\lambda < 1$).
    (a) Find the horizontal range R of the particle,
    (b) Find the maximum height H of the particle.
    (c) Show that the horizontal distance between the two vertical walls is $R\sqrt{1-\lambda}$.

13. Two straight paths, inclined to one another at $60°$, intersect at a point O. A boy A is on the path $300m$ from O, while a boy B is on the other path $400m$ from O ($A\hat{O}B = \frac{\pi}{3}$).
    Simultaneously the boys begin to run towards O, If A has constant speed $15kmh^{-1}$ and B has constant speed $12kmh^{-1}$, find the shortest distance between A and B.

14. The triangle ABC in the given figure represents a vertical cross–section through the centre of gravity of a uniform smooth wedge of mass M kg. The line AB is the line of greatest slope of the face containing it. $A\hat{B}C = \alpha$, $A\hat{C}B = \frac{\pi}{2}$ and $AB = l$. The wedge is placed with the face containing BC on a smooth horizontal floor. A particle of mass m kg is gently released from point A on the line AB.

Show that until the particle leaves the wedge, the acceleration of the wedge is $\frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$ and the acceleration of the particle relative to the wedge is $\frac{(m+M)g\sin\alpha}{M+m\sin^2\alpha}$.

Now suppose that $\alpha=\frac{\pi}{3}$ and $M = 5m$. Find that the speed of the wedge at the instant when the particle leaves the wedge.



15. $OACB$ is a parallelogram. AB and OC are diagonals. D is the midpoint of AC. OD and AB intersect at E. Let $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, $OE:ED = \lambda:1-\lambda$ and $BE:EA = \mu:1-\mu$.

i. Find $\overrightarrow{OC}$ and $\overrightarrow{OD}$. Hence write $\overrightarrow{OE}$, in terms of λ, $\underline{a}$ and $\underline{b}$

ii. Find $\overrightarrow{BA}$ and write $\overrightarrow{OE}$ in terms of μ, $\underline{a}$ and $\underline{b}$.

iii. Using the results obtained in (i) and (ii) above, find λ and μ.

iv. Hence find $\overrightarrow{OE}$.

v. Evaluate $OE:ED$ and $BE:EA$.

16. (a) In a parallelogram $ABCD$, let $AB = 4\,m$ and $AD = 2\,m$, and let $B\hat{A}D = \frac{\pi}{3}$. Also let E be the midpoint of CD. Forces of magnitude 5, 4, 3, 5 and 4 Newtons act along AB, BC, DC, DA and BE respectively, in the directions indicated by the order of the letters. Find the magnitude of the resultant force and its direction. Also, find the distance from A to the point at which the line of action of the resultant force meets AB.

(b) A non-uniform rod AB, weight W, is in equilibrium inside a smooth hollow sphere. AB subtended angle 2α with the centre of the sphere. The Centre of gravity of the rod AB is the ratio $a:b = AG:GB$ when AB makes angle θ with the horizontal. Using cot theorem, prove that $\tan\theta=\left(\frac{b-a}{b+a}\right)\tan\alpha$.

17. The ends A and D of a light inextensible string $ABCD$ are fixed and the end D is above the end A. Masses of 20 kg and m kg are connected at B and C respectively. At the equilibrium, the segment AB is horizontal and the segments BC and CD are inclined at 30^0 and 60^0 to the horizontal, respectively. Find the value of m and the tension of each segment of the string.