The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme
MYF2520 / MHF2520- Combined Mathematics 2 – Level 2
Final Examination 2023/2024

Date: 27-01-2024 **From 9:30 am. To 12:30 pm.**

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B.**

PART A

1. The position vectors of points A and B are $\underline{a}$ and $\underline{b}$ respectively. Let C be the midpoint of AB, D be a point on AB such that $AD:DB = 1:2$ and E be a point on AB such that $AE:EB = 2:1$. Find the position vectors of C, D and E.

2. If $\underline{a}$ and $(\underline{a}+\underline{b})$ are perpendicular vectors to each other, show that $|\underline{a}+\underline{b}|^2 = |\underline{b}^2 - \underline{a}^2|$.

3. If the resultant of two equal forces inclined at an angle 2θ is twice the magnitude of the resultant when they inclined at an angle 2α, show that $\cos\theta = 2\cos\alpha$.

4. $ABCD$ is a square. E and F are the midpoints of BC and CD respectively. Forces $5, 2\sqrt{5}, 5\sqrt{2}, 4\sqrt{5}, 1$ Newton act at A along AB, AE, CA, AF, AD respectively in the directions indicated by the order of the letters. Find the resultant.

5. A sphere of radius a and weight W rests on a smooth inclined plane supported by a string of length l with one end attached to a point on the surface of the sphere and other end fastened to a point on the plane. If the inclination of the plane to the horizontal is α, prove that the tension in the string is $\frac{W(a+l)\sin\alpha}{\sqrt{2al+l^2}}$.

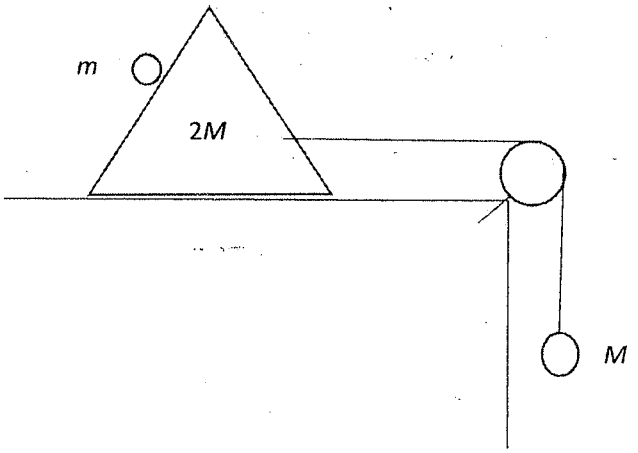
6. A motor car takes a time T to travel a distance d. In this journey, the motor car starts from rest, acquires a maximum velocity V with an acceleration of a_1 and immediately after travels under retardation a_2, come to rest.

   Show that $T = \sqrt{\frac{2d(a_1+a_2)}{a_1a_2}}$ and $V = \sqrt{\frac{2da_1a_2}{a_1+a_2}}$.

7. An observer at a point $120m$ vertically above, finds that a body passes him while ascending and again passes him while descending in five seconds. If the body is projected at ground level, find its velocity of projection.

8. The coordinates of a point on the path of a projectile relative to a point O are (x, y). If R is horizontal range, show that the angle of projection θ, is given by $\tan\theta = \frac{Ry}{x(R-x)}$.

9. As shown in the figure, a mass M suspending vertically from a string passing over a smooth pulley, pulls a wedge of mass 2M horizontally along a smooth horizontal table. A mass m is placed on the face of the inclined at 45° to the horizontal. All motions take place in a verticle plane along the line of greatest slope. Show that the acceleration of m relative to the wedge is $\frac{\sqrt{2}g(4M+m)}{(6M+m)}$.



10. The direction of motion of an enemy vessel relative to a ship sailing due north at a speed u is 45^0 east of north. For another ship travelling due south with the same speed, the enemy vessel appears to be sailing in a direction 30^0 east of north. If the true direction of sailing of the enemy vessel is θ east of north, show that $\theta = \tan^{-1}(\sqrt{3}-1)$.

PART B

11. (a) $OABC$ is a parallelogram. D is the midpoint of AB. OD and AC intersects at E. $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, $OE : ED = \lambda : 1$ and $CE : EA = \mu : 1$.

(i). Find $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{b}$. Hence write the vector $\overrightarrow{OE}$ in terms of λ, $\underline{a}$ and $\underline{b}$.

(ii). Find the vector $\overrightarrow{AC}$ and write the vector $\overrightarrow{OE}$ in terms of μ, $\underline{a}$ and $\underline{b}$.

(iii). Using the results obtained in (i) and (ii) above find λ and μ.

(iv). When OD and CB produced meet at H, find $\overrightarrow{OH}$.

(b) $\underline{a} = \underline{i} + \sqrt{3}\,\underline{j}$ where $\underline{i}$ and $\underline{j}$ have the usual meaning. $\underline{b}$ is a vector with magnitude $\sqrt{3}$. If the angle between the vectors $\underline{a}$ and $\underline{b}$ is $\frac{\pi}{3}$, Find $\underline{b}$ in the form $x\underline{i} + y\,\underline{j}$ where $x(< 0)$ and y are constants to be determined.

12. Forces $P, Q, R, P, 2P, 3P\ N$ act along the sides AB, BC, CD, DE, EF, FA respectively of a regular hexagon $ABCDEF$ of side $2a$ metres in the sense indicated by the order of letters.

(a). If the system is equivalent to a couple show that $Q = 2P$ and $R = 3P$ and calculate the moment of the couple.

(b). If the system is equivalent to a single force along AD find Q and R in terms of P.

13. (a) A rod of weight W whose centre of gravity divides its length in the ratio 2:1 lies in equilibrium inside a smooth hollow sphere. If the rod subtends an angle 2α at the centre of the sphere and makes angle θ with the horizontal, prove that $\tan\theta = \frac{1}{3}\tan\alpha$. Also find the reactions at the end of the rod in terms of W and α.

(b) A uniform ladder of weight W rests on rough horizontal ground against a smooth vertical wall. The vertical plain containing the ladder is perpendicular to the wall and the ladder is inclined at an angle α to the vertical. Prove that if the ladder is on the point of slipping and μ is the coefficient of friction between it and the ground then $\tan\alpha = 2\mu$.

14. A balloon resting on the ground starts to ascend vertically with a uniform acceleration of $\frac{g}{8}\ ms^{-2}$. After time t from the beginning, a body was released from the balloon. As a result,

the acceleration of the balloon increases to $\frac{g}{4}\ ms^{-2}$. After another time $\frac{t}{4}$, the balloon bursts and starts to fall freely under gravity. Using the same axes, draw the velocity-time graphs for the balloon and the body, g is the acceleration due to gravity. Using this graph, find

(i) the distance, the balloon has travelled when the body was released.
(ii) the time taken by the body to attain the maximum height.
(iii) the time taken by the balloon to attain the maximum height.
(iv) the time taken for the balloon and the body to posses equal velocities and that velocity.

15. A body projected with a velocity u, inclined at angel θ to the horizontal passes over a wall of height h at a horizontal distance of d. Find how hight above the wall, it passes. If the body just passes above the wall, show that $u^2 = \frac{gd^2}{2(d\ tan\theta - h)\cos^2\theta}$.

If H is the maximum height attained by the body then show that $H = \frac{d^2\tan^2\theta}{4(d\tan\theta - h)}$.

If R is the range of the partice then show also that $R = \frac{dh}{d\tan\theta - h}$.

16. Two straight roads OA and OB meet at an acute angle α. A car P moves along OA towards O with uniform speed u, while a second car Q moves along OB, away from O with uniform speed v. At $t = 0$, the car P is at a distance a from O and the car Q is at O. Find the velocity of P, relative to Q

(i) Show that the shortest distance between the cars is $\frac{av\sin\alpha}{\sqrt{u^2+v^2+2uv\cos\alpha}}$ and find the time taken to reach shortest distance.

(ii) Show that the ratio of the distances from O when they are at shortest distance is

$$v + u\cos\alpha : u + v\cos\alpha$$

17. (a) A car weight W has maximum power H. In all circumstances there is a constant resistance R due to friction. When the car is moving up a slope of $\sin^{-1}\left(\frac{1}{n}\right)$, its maximum speed is V and when it is moving down the same slope its maximum speed is $2V$. The maximum speed of the car on the level road is U. Find the maximum acceleration of the car when it is moving with speed $\frac{U}{2}$ up the given slop.

(b) Two particles A and B of masses 5kg and 3kg respectively are connected by a light inextensible string. This string passes underneath a light, smooth, movable pulley, bearing a weight of 2kg and the particles A and B rest on two rough horizontal planes. The coefficient of friction for both particles is 0.1. Find the accelerations of A and B and the tension in the string.