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The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Linear Programming- ADU5300**
Academic Year	**: 2023/24**
Date	**: 23.10.2023**
Time	**: 01.30 p.m. –03.30 p.m.**
Duration	**: Two Hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(4)** pages.
3. Answer any **(4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered as an exam offense will lead to punishment.
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script.

Answer any **FOUR** questions only

1. a) i) Write the definition of a ***convex set.*** (02 marks)

   ii) Show that $S = \{(x, y, z): z \geq x^2 + y^2\} \subset \mathbb{R}^3$ is a convex set. (08 marks)

   b) A local politician is budgeting for her media campaign. She will distribute her funds between TV ads and radio ads. She has been given the following advice by her campaign advisers;

   - She should run at least 120 TV ads and at least 30 radio ads.
   - The number of TV ads she runs should be at least twice the number of radio ads she runs but not more than three times the number of radio ads she runs.

   The cost of a TV ad is Rs. 8000 and the cost of a radio ad is Rs. 2000.

   i)Formulate a linear programming model to minimize the cost by clearly defining the decision variables. (05 marks)

   ii) Use graphical method to determine which combination of TV and radio ads she should choose to minimize the cost of her media campaign. (10 marks)

   [Total marks 25]

2. a) Consider the following table of a Simplex method. Suppose that the index row elements should be greater than or equal to zero to reach the optimal condition.

Basic variables	x_1	x_2	x_3	s_1	s_2	s_3	**RHS value**
x_2	2	1	-1	1	0	0	2
s_2	4	0	4	1	1	0	8
s_2	2	0	2	-1	0	1	4
Z	3	0	-3	2	0	0	4

Write down the next pivoting of the Simplex method to reach the optimal solution by avoiding cycling. (10 marks)

b) Solve the following linear programming problem using the Revised Simplex method:

$$\begin{aligned} &Minimize\ Z = -6x_1 - 5x_2 \\ &subject\ to\ 4x_1 + x_2 \leq 800 \\ &\qquad 2x_1 + 3x_2 \leq 900 \\ &\qquad x_1 \leq 180 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

(15 marks)

[Total marks 25]

3. a) When applying the Big-M method to a maximization linear programming problem the following table is obtained, where x_1, x_2,and x_3 are decision variables, s_1, s_2 and s_3 are surplus or slack variables and, a_1 and a_2 are artificial variables. Suppose that the index row elements should be greater than or equal to zero to reach the optimal condition. What can you conclude from this table? Explain your answer.

BV	x_1	x_2	x_3	s_1	s_2	s_3	a_1	a_2	RHS
x_3	$1/2$	$1/2$	1	$1/2$	0	0	0	0	2
a_1	$-3/2$	$-1/2$	0	$-1/2$	-1	0	1	0	2
a_2	$-1/2$	$-1/2$	0	$-1/2$	0	-1	0	1	1
z	$-\frac{1}{2} + 2M$	$-\frac{1}{2} + M$	M	$\frac{1}{2} + M$	M	M	0	0	$2 - 3M$

(05 marks)

b) Consider the following linear programming problem:

$$\begin{aligned} &Maximize\ Z = 4x_1 + 3x_2 \\ &Subject\ to\ \ x_1 + x_2 \leq 50 \\ &\qquad x_1 + 2x_2 \geq 80 \\ &\qquad 3x_1 + 2x_2 \geq 140 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Solve the given problem using the Two-phase Simplex method.

(20 marks)

[Total marks 25]

4. a) Write the dual of the following primal problem:

$$\begin{aligned} Minimize\ Z = {} & x_1 - x_2 - x_3 \\ Subject\ to\ & -3x_1 - x_2 + x_3 \leq 3 \\ & 2x_1 - 3x_2 - 2x_3 \geq 4 \\ & x_1 - x_3 = 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(10 marks)

b) Solve the following linear programming problem using dual simplex method:

$$\begin{aligned} Minimize\ Z = {} & 2x_1 + x_2 \\ Subject\ to\ & 3x_1 + x_2 \geq 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \geq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(15 marks)
[Total marks 25]

5. An IT company has five software engineers, and it needs to develop five computer programs. The head of the company estimates the computer time (in hours) required by the engineers to develop the programs as given below:

Program / Engineer	I	II	III	IV	V
E1	9	22	58	11	19
E2	43	78	72	50	63
E3	41	28	91	37	45
E4	74	42	27	49	39
E5	36	11	57	22	25

i) Formulate a mathematical model to find an optimum assignment by clearly defining the decision variables. (05 marks)

ii) Use **Hungarian Algorithm** to solve the formulated model in part (i). (15 marks)

iii) Write the optimum assignment. (03marks)

iv) Find the minimum time required to develop the five computer programs. (02 marks)

[Total marks 25]

6. i) Explain degeneracy in a transportation problem when formulated as a linear programming problem? (05 marks)

ii) A company has three factories, F1, F2, and F3 and four warehouses, W1,W2, W3and W4. The shipping cost (in thousand rupees) from factories to the warehouses are shown below in the form of a matrix:

	W1	**W2**	**W3**	**Factory capacity in units**
F1	5	4	3	250
F2	8	4	3	300
F3	9	7	5	300
Warehouse requirement in units	300	200	200	

a) Formulate the above transportation problem as a linear programming model in order to minimize the shipping cost. (03 marks)

b) Find the initial basic feasible solution (IBFS) using North-West Corner method. (05 marks)

c) By using the IBFS obtained in part (b), apply the transportation algorithm to find the optimal manufacturing cost. (10 marks)

d) Determine the minimum transportation cost to satisfy each Factory capacity and each Warehouse requirement. (02 marks)

[Total marks 25]

********* END OF QUESTION PAPER *********