The Open University of Sri Lanka

B.Sc. /B.Ed., Continuing Education Degree Programme

Applied Mathematics – Level 05

ADU5302/ADE5302 – Mathematical Methods

No Book Test (NBT) – 2023/2024

DURATION: ONE HOUR

Date: 02.09.2023. **Time: 14:30h – 15:30h**

ANSWER **ALL** QUESTIONS.

1.The Gamma function denoted by $\Gamma(p)$ corresponding to the parameter p is defined by the improper integral $\Gamma(p)=\int_0^\infty e^{-t}t^{p-1}dt, \quad (p>0)$.

(a) Using the result $\Gamma(p+1)=p\Gamma(p)$, Compute each of the following:

(i) $\dfrac{\Gamma(4).\Gamma(3.5)}{\Gamma(5.5)}$

(ii) $\Gamma(-4.3)$

(b) Evaluate $\int\limits_0^\infty e^{-h^2x^2}dx$;where h is a positive integer.

2. The Beta function denoted by $\beta(p,q)$ is defined by $\beta(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}dx$, where $p>0$ and $q>0$ are parameters.

(a) Compute $\int_0^{\frac{\pi}{2}}\left(\dfrac{\sqrt[3]{\sin 8x}}{\sqrt{\cos x}}\right)dx$

(b) Use Beta function to evaluate $I=\int\limits_0^1 x^4\sqrt{1-x^2}\,dx$.

3. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x) = x^p \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+p} . m! \Gamma(p+m+1)}.$$

(i) Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$,

(ii) Evaluate $\int J_5(x)dx$.

(iii) Show that $\frac{d}{dx}\left[J_n^2 + J_{n+1}^2\right] = 2\left[\frac{n}{x}J_n^2 - \frac{n+1}{x}J_{n+1}^2\right]$.

(Hint: You may use the following recurrence relations, if necessary, without proof.)

(i) $\frac{d}{dx}\{x^p J_p(x)\} = x^p J_{p-1}(x)$

(ii) $\frac{d}{dx}\{x^{-p} J_p(x)\} = -x^{-p} J_{p+1}(x)$.

(iii) $\frac{d}{dx}\{J_p(x)\} = J_{p-1}(x) - \frac{p}{x}J_p(x)$ or $xJ_p'(x) = xJ_{p-1}(x) - pJ_p(x)$

(iv) $J_p'(x) = \frac{p}{x}J_p(x) - J_{p+1}(x)$

(v) $J_p'(x) = \frac{1}{2}\{J_{p-1}(x) - J_{p+1}(x)\}$

(vi) $J_{p-1}(x) + J_{p+1}(x) = \frac{2p}{x}J_p(x)$