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The Open University of Sri Lanka

Advanced Certificates in Science – Level 2 Part 1

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2520 - Mathematics 2—Paper I

Date: 27th December 2021 **Time: 09.30 am – 12.30 pm**

Instructions

You are allowed to use non-programmable calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including one question from Part A & part B.

Part A – Trigonometry (Answer one question only)

(1) (a) By using the general solutions, solve the following equations of x, where $0 \leq x < 2\pi$.

(i) $\sin x = -\frac{1}{2}$ (ii) $\tan x = -1$ (iii) $\cos 2x = \frac{\sqrt{3}}{2}$

(b) Find the most general value of θ that satisfies both equations.

$$\sin\theta = -\frac{\sqrt{3}}{2} \text{ and } \tan\theta = \sqrt{3}$$

(c) Find the general solutions of the following trigonometric equations.

(i) $\cos 2x + \sin x - 1 = 0$ (ii) $\sin\theta + \sin 3\theta = \cos\theta + \cos 3\theta$

(2) (a) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form of $R\sin(\theta + \alpha)$, where R and α are real. Hence, find the general solutions of the equation $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$.

(b) Solve the equation $2\cos\theta\cos 2\theta + \sin 2\theta = 2(3\cos^3\theta - \cos\theta)$, for the values of θ within the range of $(0 < \theta < 2\pi)$.

(c) If $\tan\frac{\theta}{2} = t$, then show that $\cos\theta = \frac{1-t^2}{1+t^2}$ and $\sin\theta = \frac{2t}{1+t^2}$. Hence find the general solution of the equation $\sqrt{3}\sin\theta + \cos\theta = 1$.

Part B – Coordinate Geometry (Answer one question only)

(3)(a) Find the equation of the straight lines which bisects the angles between the lines $2x + y - 3 = 0$ and $3x + y - 3 = 0$. Show that those lines are right angles to each other.

(b) Find the equation of straight line with equal intercepts of coordinate axes and passing through the intersection point of lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$.

(4) In a triangle ABC, equations of AB and AC are $2x - y - 1 = 0$ and $x - 2y + 1 = 0$ respectively. The midpoint of BC is $(-2, -2)$. Find,

(i) the coordinates of the midpoint of AC

(ii) the equation of BC

(iii) the area of the triangle ABC

(iv) the coordinates of the centroid of the triangle ABC

Part C – Algebra (Answer three questions only)

(5) (a) The polynomial $f(x) = x^4 + 2x^3 + ax^2 + bx + c$ is divided by $x^2 + x - 2$. When is divided by $(x + 1)$, the remainder is -8. Find the values of a, b and c.

(b) Express the following rational function as partial fractions.

$$\frac{9x^2 + 35x + 31}{(x+2)(x^2 + 3x + 2)}$$

(c) Sketch the graph of the rational function, $y = \frac{x-1}{x+2}$.

(6) (a) Solve the following inequalities and indicate it on a number line.

(i) $3x - 5 \leq 3 - x$ (ii) $x^2 - 3x + 2 > 0$

(iii) $-2x^2 + 5x + 12 = 0$ (iv) $|x - 4| < 3$

(b) Sketch the following inequalities and get the solution.

(i) $|x + 2| > 2$ (ii) $|x^2 - x| > 6$

(7) (a) Find the sum of the arithmetic series $15, 9, 3, \cdots\cdots, -45$.

(b) S_n denotes the sum of a geometric series for n terms. If $33S_5 = S_{10}$, then find the first term and the common ratio of the series.

(8) (a) In how many ways can 5 boys and 3 girls seat themselves in a row if

(i) the three girls are to seat next to each other.

(ii) the three girls are not to seat to each other.

(b) In how many ways can 4 boys and 4 girls be arranged in a circle so that the boys and girls occupy alternate places.

(c) In a set of 32 cards there are 8 blacks, 8 red, 8 blue and 8 green cards. Cards of the same colour are all different.

(i) Find the number of different ways in which 3 cards may be selected at random from the set.

(ii) Find also the number of selections in (i) for which the cards, are not all of different colour.

(9) (a) Find the coefficient of x^5 and x^{10} and the term independent of x in the binomial expansion of $\left(\frac{x^2}{2} - \frac{3}{x^3}\right)^{15}$.

(b) When $x = 2$, find the greatest term in the expansion of $(3 + x)^9$.

END.

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Advanced Certificates in Science – Level 2 Part 1

Final Examination – 2020/2021

Duration: Three (03) hours

MHF2520 - Mathematics 2–Paper II

Date: 3rd January 2022 Time: 9.30 am – 12.30 pm

Instructions

You are allowed to use non-programmable calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions.

Part A – Trigonometry

(Answer only one question Part A)

(1)(a) Prove that,

$$\cos(45^0 - A)\cos(45^0 - B) - \sin(45^0 - A)\sin(45^0 - B) = \sin(A + B)$$

(b) Simplify the following expressions.

(i) $\sin 2x + \sin 6x + \sin 5x + \sin 3x$ (ii) $\dfrac{\cos 75^0 - \cos 15^0}{\sin 75^0 + \sin 15^0}$

(c) Solve the following equation for x.

$$\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}(3x) - \tan^{-1}(x)$$

(2) (a) Sketch the graph of the following functions.

(i) $y = \sin\left(2x + \frac{\pi}{3}\right)$ (ii) $y = cos\left(2x - \frac{\pi}{3}\right)$ (iii) $y = \tan\left(\frac{x}{3} + \frac{\pi}{6}\right)$

(b) Prove that,

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$$

Part B – Statics

(Answer only two questions from Part B)

(3)(a) Express the Lami's theorem and explain with a diagram.

(b) One end of an inextensible string of length 5 m is fixed to a point A and the other end of the string is fastened to a small object of weight 1 kg. The object is pulled aside by a horizontal force, until it is 3 m from the vertical through A. Find the magnitudes of the tension of the string and the horizontal force ($g = 10\ ms^{-2}$).

(4) (a) A uniform sphere of radius a and weight w rests in contact with a smooth vertical wall and is supported by an inextensible string of length l. The string is connected to center C of the sphere and a point B on the wall. Find the tension in the string and the reaction by the wall on sphere.

(b) A smooth peg is fixed at a point P at distance a from a smooth vertical wall. A uniform rod AB of length $6a$ and weight W is in equilibrium resting on the peg with the end A in contact with the wall.

Taking θ to be the angle made by the rod AB with the horizontal draw a triangle of forces, representing forces acting on the rod. Find the reaction at P, in terms of W and θ. Show that $3\cos^3\theta = 1$.

(5) AB and BC are two uniform rods, of equal length $2a$ and of weights W and $2W$ respectively. They are smoothly hinged together at B and also hinged at A and C to a fixed horizontal beam. The rods are in equilibrium in a vertical plane with B below AC and $C\hat{A}B = \alpha$.

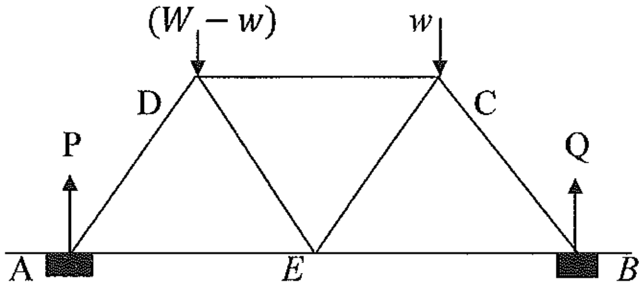
(i) Show that the horizontal component of the reaction of the hinge at B is $\frac{3}{4}W\cot\alpha$, and find the vertical component of this reaction.

(ii) If, further, the lines of action of the reactions at A and C are perpendicular to each other, show that $\tan\alpha = \frac{3}{\sqrt{35}}$.

(6) A framework is made up of seven light rods of equal length freely jointed, as shown in figure. A and B rest on smooth supports and there are loads of $(W - w)$ at D and w at C. Show that the reaction on the framework at A is

$P = \frac{3W}{4} - \frac{w}{2}$. Given that $W > 2w$, draw a suitable stress diagram using Bow's notation and find the stresses in the rods AE, DE and DC.

Indicate whether they are tension or thrusts. Show that the stress in DC is independent of w.



Part C – Dynamics

(Answer only two questions from Part C)

(7)(a) Ship A is moving due North at $20\ kmh^{-1}$. At the same instant the ship B $2\ km$ from East to ship A and moving due West at $15\ kmh^{-1}$. Find the velocity of A relative to B and shortest distance between two ships.

(b) A car of mass $1000\ kg$ is towing a caravan of mass $600\ kg$ along a horizontal road. Given that the driving produced by the engine is $400\ N$ and that there is no resistance to motion. Find the tension in the tow-bar and the acceleration of the car.

(8)(a) A particle of mass $200\ g$ resting on a smooth plane inclined at 30^0 to the horizontal has a light string attached to it with the other end having a mass of $400\ g$. The string passing over a smooth pully at the top of the plane is held and gently released. Find which particle will go down and find its acceleration ($g = 9.81\ ms^{-2}$).

(b) A sphere of mass m moving with velocity u collides directly with an identical sphere which is stationary. Find the velocities of the spheres after collision. When the coefficient of restitution is 0.5, find the loss of kinetic energy.

(9) A car of weight W has maximum power H. In all circumstances there is a constant resistance R due to friction. When the car is moving up a slop 1 in n $\left(\sin^{-1}\left(\frac{1}{n}\right)\right)$. Its maximum speed is v and when it is moving down the same slope it maximum speed is $2v$. Find R in terms of W and n.

END.