

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme



Department	: Mathematics
Level	: 03
Name of the Examination	: Final Examination
Course Title and - Code	: Differential Equations – ADU 3302/APU1142/ADE3302
Academic Year	: 2019/2020
Date	: 05.11.2020
Time	: 09.30 am – 11.30 am
Duration	: 02 hrs

General Instructions

1. Read all instructions carefully before answering the questions.
2. This paper consists of **TWO** sections, Section **A** and Section **B**. Section **A** is compulsory and it consists of SIX Structured Essay Questions. You should answer in the space under each question.
3. Section **B** consists of **FIVE** essay type questions and answer only **THREE** of them. Each question in Section **B** carries 100 marks.
4. This paper consists of 05 pages.
5. Answer for each question in section **B** should commence from a new page.
6. Involvement in any activity that is considered as an exam offence will lead to punishment
7. Use a blue or black ink pen to answer the questions.
8. Clearly state your index number in your answer script.
9. At the end of the exam, attach Section **A** to the answer booklet and hand over to the supervisor.

SECTION A

1. Answer all the questions in this section in the space provided.

(a) Solve the following initial value problem:

$$\frac{dx}{dt} = \cos t + \sin t, \quad x(\pi) = 1.$$

(b) Find the general solution of the equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 4y = 0$.

- (c) The velocity v of a moving object is given as $v = 32t - 2$ and the position of the object when $t = \frac{1}{2}$ is 4. Find the expression for the position of the object at time t .

- (d) Find the integrating factor of the equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$.

(e) Find the UC set of the function $f(x) = e^x x^2 \cos(x+2)$.

(f) Find the value of the constant p such that the equation $(x^2 - 2xy)dx + (px^2 + 2y)dy = 0$ is exact.

SECTION B

Answer **THREE** Questions **ONLY** from this section.

2. (a) Consider the differential equation $dy - (x + y + 3)^2 dx = 0$.

(i) Is the given equation homogeneous? Justify your answer.

(ii) Using a suitable substitution find the solution of the given equation.

(b) A first order linear differential equation is given as

$$(P(x)y - Q(x))dx + dy = 0.$$

(i) Check whether the given equation is exact.

(ii) If the equation is not exact, then make it to an exact differential equation.

(iii) If $P(x) = -3$ and $Q(x) = 9$, find the solution of the equation.

3. Consider the given differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 50x^3 - 36x^2 - 63x + 18$.

a) Find the solutions y_c of the homogeneous equation corresponds to the given equation.

b) Form the UC sets for the UC functions of the non-homogeneous term corresponds to the given equation. Generate the revised sets after performing, omitting and/or revising the sets.

c) Using the revised sets obtained in the part (b), find the particular solution of the equation.

d) Write down the general solution of the given equation.

4. An object of mass m is dropped from a height 600ft with no initial velocity. The force due to air resistance is equal to the value $\frac{v}{4}$, where v is the velocity of the object (ft/sec).

Assume $m = 2$ and $g = 32 \text{ ft/sec}^2$.

a) Find the expression for the velocity of the object at any time t .

b) Find the limiting velocity for the object.

c) Find the expression for the position of the object at any time t .

d) Find the time required for the object to hit the ground. (Hint: the negative exponential term will be negligible for any larger value of t .)

5. (a) (i) Suppose $f(D) = a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n$, where a_0, a_1, \dots, a_n are constants.

If k is a constant show that $f(D)e^{kx} = f(k)e^{kx}$.

(ii) Find the particular solution of the equation $y'' + 3y' - 9y = e^{-3x} + 10$ using D-operator method.

(b) Consider the differential equation $\frac{d^2 y}{dx^2} + \frac{\cos x}{x^3} \frac{dy}{dx} + \frac{e^x}{(x-1)^2} y = 0$.

Find the singular points of the equation. Then identify the regular singular points and irregular singular points.

6. Suppose that an investment at time t is P and the rate change of the values of the investment is $\frac{dP}{dt} = rP$ where r is the interest rate for unit time t . Assume that the interest rate is compounded continuously and the value of the initial investment is P_0 .

a) Show that $P(t) = P_0 e^{rt}$.

b) A person deposits Rs. 100 in a savings account that accumulates interest compounded continuously. The interest rate is constant 5 percent per annum for the first four years and a constant 8 percent per annum for the last year. Using part (a), write down the principle value of the saving at the end of 4 years. Hence find the amount in the saving account at the end of 5 years. [Assume that t is measured in years.]

c) Suppose that the person withdraws money from the account at a constant rate k .

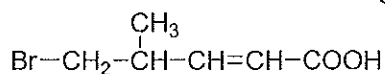
Modify the equation $\frac{dP}{dt} = rP$ according to the given facts and find the expression for

$P(t)$.

13. Of the following hydrides of Group 16 (or Group VIA) elements, the one with the lowest boiling point is

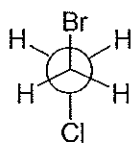
- (1) H_2O (2) H_2S (3) H_2Se (4) H_2Te (5) H_2Po

14. What is the IUPAC name of the following compound?

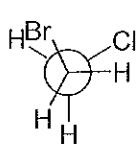


- (1) 1-bromo-2-methylpent-3-en-5-oic acid
 (2) 2-methyl-1-bromopent-3-en-5-oic acid
 (3) 4-methyl-5-bromo-2-pentenoic acid
 (4) 5-bromo-4-methyl-2-pentenoic acid
 (5) 4-bromomethyl-2-pentenoic acid

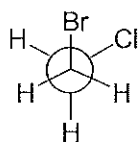
15. Select the correct statement regarding the following conformations A- F.



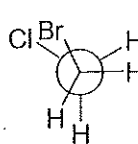
A



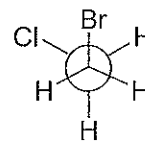
B



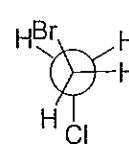
C



D



E



F

- (1) Stabilities of conformations **A** and **C** are the same.
 (2) All eclipsed conformations have the same stability.
 (3) **D** has the least stability among all.
 (4) Stability of **C** is less than that of **E**.
 (5) Conformations **B** and **F** have the same stability.

16. Consider the following statements regarding the chlorination of CH_4 in the presence of sunlight.

- (a) Formation of chlorine free radicals is the chain initiation step.
 (b) In the presence of excess chlorine, a mixture of chlorinated products is formed.
 (c) CH_3CH_3 can be formed as a product.

True statement/s is/are,

- (1) (a) only. (2) (b) only. (3) (c) only. (4) (a) and (b) only (5) All (a), (b) and (c)

17. Select the wrong statement regarding petroleum refining.

- (1) Crude petroleum is mainly a mixture of hydrocarbons.
 (2) Fractions obtained by distillation of crude petroleum are pure alkanes.
 (3) Thermal cracking of petroleum leads to unbranched hydrocarbons
 (4) Petroleum cracking can be done by using a catalyst at high temperatures.
 (5) Branched smaller alkanes are most suitable to be used as gasoline.

18. Which of the following processes gives no heat exchange between the surroundings and the system?

- (1) adiabatic (2) isothermal (3) isobaric (4) isochoric (5) isenthalpic

19. Which of the following is true of an isothermal reversible gas expansion?

- (1) no heat is exchanged between the system and the surroundings
- (2) the temperature of the surroundings remains constant
- (3) the system does positive work on the surroundings
- (4) the total heat absorbed by the system = total work done by the system
- (5) work done by the system is zero

20. Which of the following expression cannot be used for a reversible, adiabatic transformation in an ideal gas?

- (1) $PV^\gamma = \text{constant}$
- (2) $PV = nRT$
- (3) $TV^{\gamma-1} = \text{constant}$
- (4) $\Delta H = nC_{p,m}\Delta T$
- (5) $q = \Delta U$

21. The SI unit for molar entropy is,

- (1) $\text{JK}^{-1}\text{mol}^{-1}$
- (2) JK^{-1}
- (3) Jmol^{-1}
- (4) JK
- (5) JKmol^{-1}

22. Which thermodynamic quantity predict the spontaneity of a reaction in a closed system ?

- (1) ΔS for the reaction at constant T and P
- (2) ΔU for the reaction at constant P
- (3) ΔH for the reaction at constant T
- (4) ΔG for the reaction at constant T and P
- (5) both ΔS and ΔG for the reaction at constant T and P

23. For a spontaneous process which one of the following expression will not apply

- (1) $dS_{\text{sys}} > dq/T$
- (2) $dS_{\text{universe}} > 0$
- (3) $dG_{p,T} > 0$
- (4) $dU = dq + dw$
- (5) $dw = -P_{\text{ex}}dV$

24. According to IUPAC conventions, the cell reaction corresponding to a cell diagram

- (a) Is written in such a way so that the oxidation takes place at the electrode on the left-hand side.
- (b) Is written in such a way so that the cathodic half reaction takes place at the electrode on the right-hand side.
- (c) Does not have to be spontaneous.

The correct statements, out of (a), (b) and (c) above, are

- (1) (a) and (b) only
- (2) (a) and (c) only
- (3) (b) and (c) only
- (4) All (a), (b) and (c)
- (5) None of the **answers** (1), (2), (3) or (4) is correct

25. A student prepared a solution by dissolving NaBr and Na_2SO_4 in distilled water. The concentration of NaBr and Na_2SO_4 $0.100 \text{ mol dm}^{-3}$ and $0.400 \text{ mol dm}^{-3}$ respectively. What is the ionic strength of the solution in mol dm^{-3} .

- (1) 1.7
- (2) 1.5
- (3) 1.3
- (4) 2.6
- (5) 3.0

26. The activity, a_x of an ionic species X in solution is given by $a_x = \gamma_x \left(\frac{C_x}{C^0} \right)$;

- (a) C^0 is always unity irrespective of the units
- (b) C_x is the molar concentration of the species X
- (c) the value of γ_x depends on the particular ionic species

The correct statements out of (a), (b) and (c) above are,

- (1) (a) and (b)
- (2) (a) and (c)
- (3) (b) and (c)
- (4) All (a), (b) and (c)
- (5) Only (b)

27. When a Galvanic cell is connected to a load

- (a) oxidation takes place at the spontaneous cathode
- (b) electrons in the external circuit flows from spontaneous anode to spontaneous cathode
- (c) spontaneous cathode is at a higher electric potential than the spontaneous anode

The correct statements out of (a), (b) and (c) above are,

- (1) (a) and (b) (2) (a) and (c) (3) (b) and (c) (4) All (a), (b) and (c) (5) Only (a)

28. Consider the following statements:

- (a) Haber Process $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \xrightarrow{\text{Pt catalyst}} 2\text{NH}_3(\text{g})$ with a Pt catalyst is an example of a heterogeneous catalysis
 - (b) Refrigeration is an example illustrating the effect of temperature on rates of reaction.
 - (c) In accordance with Arrhenius equation, a plot of $\ln k$ (where k is the rate constant) against T (temperature in Kelvin) is expected to be linear.
 - (d) Half-life of a reaction of a first order reaction is independent of the initial concentration.
- Of the above statements,

- (1) only (a), (b), and (c) are correct (2) only (b), (c) and (d) are correct
 (3) only (c), (d) and (a) are correct (4) only (d), (a) and (b) are correct
 (5) all of (a), (b), (c) and (d) are correct

29. The relationship between two variables, k and T is given by the logarithmic expression $\ln k = Q \left(\frac{1}{T} \right) + \ln A$ (Q and A are constants)

The corresponding exponential form of the equation is

- (1) $k = \frac{1}{A} e^{\frac{Q}{T}}$ (2) $k = -\frac{1}{A} e^{\frac{Q}{T}}$ (3) $k = \frac{1}{A} e^{\frac{-Q}{T}}$ (4) $k = A e^{\frac{Q}{T}}$ (5) $k = \frac{1}{A} e^{\frac{T}{Q}}$

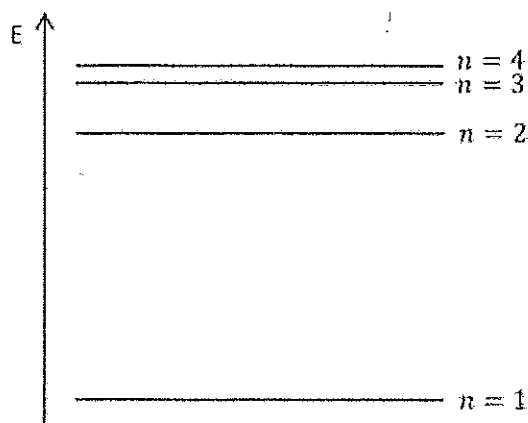
30. $\text{A} \rightarrow \text{P}$ is a first order reaction whose rate constant is $2.0 \times 10^{-4} \text{ s}^{-1}$ and the initial concentration of $\text{A} = 0.30 \text{ mol dm}^{-3}$. The half-life of this reaction (in min) is, approximately,

- (1) 58.2 (2) 38.7 (3) 77.4 (4) 3.49×10^3 (5) 2.33×10^3

Section II - Answer All Questions

- 01.(a) In the hydrogen atom, an electron absorbs a photon and moves from the $n = 2$ to $n = 3$ energy level, and then drops back to the ground level. Atoms **fluoresce** when they move from an excited energy level to the ground state releasing energy in the form of light.

- (i) Draw an arrow that represents the transition of the electron **during fluorescence** on the energy diagram below.



- (ii) Will the wavelength of the photon emitted in the above case be higher, lower or the same as the wavelength of the photon absorbed by the electron **initially**? Briefly explain your answer.
- (iii) Calculate the energy of a photon of light with wavelength 500.0 nm.
- (iv) Which electron transition/s between the energy levels of hydrogen causes the emission of a photon/s in the visible region of the Electromagnetic spectrum?
- (v) How would you indicate the electron transition when the hydrogen atom undergoes ionization?

(50 marks)

- (b)(i) Explain the difference between an orbit and an orbital.

- (ii) Give the numerical values of n , l and m quantum numbers corresponding to each of the following orbital designations by filling the table:

Orbital designation	n value	l value	m value
2s			
3p			

(30 marks)

- (c) Explain why

- (i) Li has a larger first ionization energy than Na
 (ii) Graphite conducts electricity whereas diamond does not.

(20 marks)

02. (a) Draw a fully labelled Born- Haber cycle for the formation of $\text{MgI}_2(\text{s})$. Write the expression for the lattice energy of $\text{MgI}_2(\text{s})$ using the energy terms in the cycle.

(20 marks)

(b) For each of the molecules, CHCl_3 and CH_4 , indicate using arrows, the direction of polarities of bonds and the net dipole moment, if any.

(20 marks)

(c)(i) Using the concept of hybridization, predict the geometry of BCl_3 and PCl_3 .

(ii) Draw the resonance structures for CO_3^{2-} .

(30 marks)

(d)(i) Draw the molecular orbital energy diagram of O_2 . Calculate the bond order of O_2^{2-} .

(ii) Draw and label the molecular orbitals formed by the overlap of two p_y orbitals and two p_z orbitals (z- axis is the inter- nuclear axis).

(30 marks)

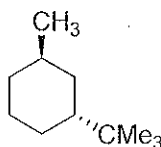
03. Answer **only five parts from parts (a) to (f)**. If more than five parts are answered, only the first five in the order of answering will be marked.

(a) Determine the configuration of the double bonds in the following compounds as E or Z. Show the priority order of the groups according to Cahn-Ingold-Prelog rules clearly.



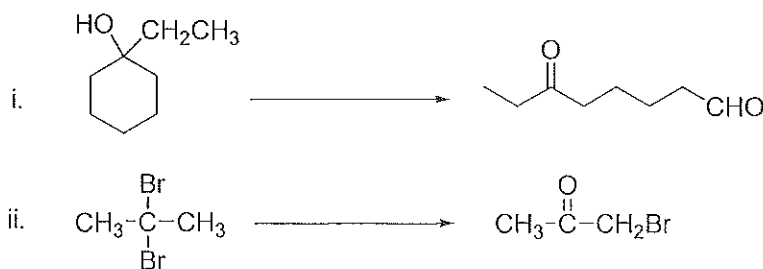
(20 marks)

(b) Draw the two chair conformations of the following compound and state giving reasons which conformation is more stable.



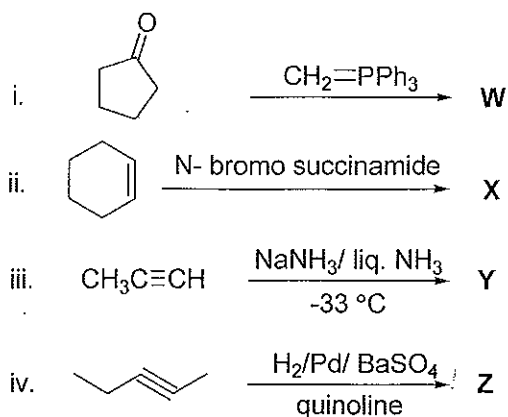
(20 marks)

(c) Giving suitable reagents and conditions, show how you carry out the following conversions.



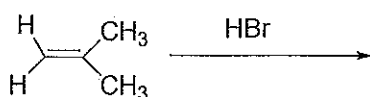
(20 marks)

(d) Give the structures of the major products of the following reactions.



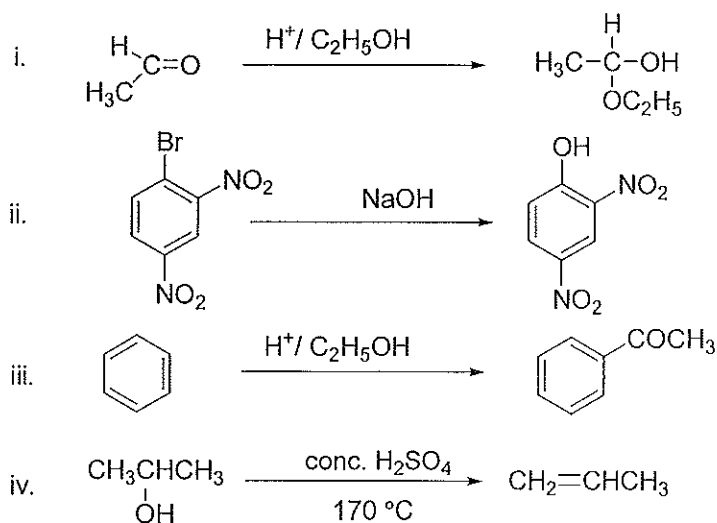
(20 marks)

(e) Give the product and the mechanism of the following reaction.



(20 marks)

(f) Classify the following reactions as elimination, addition, substitution or rearrangement reactions.



(20 marks)

04(a). Write down mathematical expressions for the following using the standard symbols.

(i) root mean square speed (ii) mean speed (iii) mean translational energy

(15 marks)

- (b). 3 mols of an ideal gas at 27°C and 4 atm pressure undergo an isothermal expansion to half the initial pressure. Calculate the work done on the gas by the surrounding when this expansion takes place
- reversibly
 - irreversibly
- against an external pressure of 2 atm.

(35 marks)

- (c) (i) Define "Thermal capacity" (C) of a substance using a mathematical expression.
 (ii) Indicate to what type of system the concept of thermal capacity will apply.
 (iii) Starting from the first law of thermodynamics show that the heat change in an isochoric (constant volume) process is equal to the internal energy change, (ΔU) provided only expansion work is possible.

(25 marks)

- (d). 1000 moles of water vapour are condensed to liquid water at 373K under standard atmospheric pressure. The standard enthalpy change for the condensation is -40 kJ mol^{-1} . Calculate the following change in properties accompanying this process.
- Entropy, ΔS
 - Gibbs free energy, ΔG

(25 marks)

05(a). Define the following as applied in studying Galvanic Cells.

- Electromotive force of a Galvanic cell
- Negative terminal of a Galvanic cell
- Electrode potential of an electrode.

(24 marks)

- (b) A student prepared a Galvanic cell by electrically connecting the ionic phases of two metal – metal ion electrodes, $\text{X(s)} \mid \text{X}^{2+}(\text{aq})$ and $\text{Y(s)} \mid \text{Y}^{3+}(\text{aq})$, using a salt bridge. Under the experimental conditions the electrode potentials of the electrodes are given by $E_{\text{X(s)} \mid \text{X}^{2+}(\text{aq})} = 1.342\text{ V}$ and $E_{\text{Y(s)} \mid \text{Y}^{3+}(\text{aq})} = 3.450\text{ V}$.

- Giving reasons, state the spontaneous anode of the above given cell.
- Giving reasons, write down the spontaneous anode, cathode and cell reactions of the above cell.
- What is the charge number of the cell reaction written above?
- Using standard notation, draw a cell diagram for the above cell whose cell reaction is non-spontaneous.
- Giving reasons assign an emf to the cell diagram you have drawn (under the conditions the student prepared the cell.)

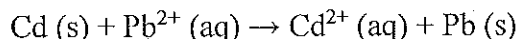
(40 marks)

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(c) (i). Write down the relationship between the Gibbs free energy and the emf assigned to a reaction and identify all the parameters in it.

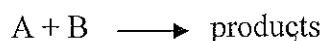
(ii) Given that, $E^0(\text{Cd}^{2+} | \text{Cd}) = -0.43 \text{ V}$ and $E^0(\text{Pb}^{2+} | \text{Pb}) = -0.126 \text{ V}$.

Calculate the E^0 and ΔG^0 for the cell reaction,



(36 marks)

06. (a) A hypothetical elementary reaction is of the form



- (i) Write down the rate expression for the above reaction using the standard symbols
- (ii) Determine the **SI units** of the rate constant.
- (iii) Assuming that this reaction is carried out with an excess amount of B relative to A, **derive** an expression for the **pseudo** rate constant (k^*) of the reaction in term of the concentration of A, [A] at time, t and its initial concentration of $[A_0]$

(36 marks)

(b) A certain reaction $[\text{A}] \longrightarrow [\text{P}]$ is found to follow the differential rate law,

$$-\frac{d[\text{A}]}{dt} = k$$

Given $A_0 - A = kt$ (standard symbols used) and 25% of A undergoes reaction in 30 minutes, what would be the concentration of A at the end of one hour(initial concentration of A is $4.0 \times 10^3 \text{ mol m}^{-3}$).

(24 marks)

(c) How are the two terms Activation Energy and Catalyst related to each other?

(12 marks)

(d) $-\frac{d[\text{A}]}{dt} = k [\text{A}]^p [\text{B}]^q$ is the general expression that represents the rate equation for a reaction of the form $\text{A} + \text{B} \longrightarrow \text{P}$

With the aid of the relevant mathematical expression, briefly outline the principle of the **Initial Rate method** for the determination of order with respect to A (experimental details **NOT** necessary).

(28 marks)

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