00109

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 5
Name of the Examination	: Final Examination
Course Title and - Code	: ADU5304/APU3146 – Operational Research
Academic Year	: 2019/20
Date	: 31/12/2020
Time	: 9.30 a.m – 11.30 a.m
Duration	: 2hrs

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **6** questions in **7** pages.
3. Answer any **4** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered as an exam offense will lead to punishment
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script

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Question 01

(a) Explain the assumptions underlying game theory.

(b) Discuss pure and mixed strategies.

(c) Explain two-person zero-sum game.

(d) Two companies, *Alpha* and *Beta*, are trying to choose spending on research and development. They can choose either high spending or low spending. If both companies choose low spending, *Alpha* will earn 10 million per year. If both companies choose high spending, *Beta* will earn 5 million per year. If one chooses high and the other low, the company that chooses high spending will earn 20 million, while the company that chooses low will earn 2 million. The companies must make their decision without knowledge of the other's action.

(i) Construct the payoff matrix with respect to the company *Alpha*.

(ii) Is there a saddle point? Justify your answer.

(iii) Determine the optimal strategies for *Alpha* and *Beta*.

Question 02

(a) Consider the following payoff matrix for 2×2 two-person zero-sum game which does not have any saddle point:

Player B

Player A		B_1	B_2
	A_1	a_{11}	a_{12}
	A_2	a_{21}	a_{22}

(i) Write down the formulas for optimum mixed strategies of Player A and Player B and the value of the game.

(ii) Prove that if a fixed positive number M is added to each element of the above pay-off matrix, then the optimal strategies remain unchanged while the value of the game increases by M.

(b) There are two major soft drink companies *MyCola* and *Sunquick*. *MyCola* is the market leader and *Sunquick* has developed several marketing strategies to gain a larger percentage of the market now belonging to *MyCola*. The following payoff matrix shows the gains for *Sunquick* and the losses for *MyCola* given the strategies of each company:

Sunquick	*MyCola*		
	A	*B*	*C*
I	10	9	3
II	4	7	5
III	6	8	-4

(i) Determine the mixed strategy for each company.

(ii) Find the expected market share gains for *Sunquick* and losses for *MyCola*.

Question 03

(a) Briefly explain the following terms:

(i) Queue discipline

(ii) Service mechanism

(iii) Service channel

(b) Patients arrive in a Poisson distribution at the Government hospital for emergency service at the rate of one every hour. Currently, only one emergency case can be handled at a time. Patients spend on average of 20 minutes receiving emergency care. The service time is found to have an exponential distribution.

(i) What is the probability that a patient arriving at the hospital will have to wait?

(ii) Find the average length of the queue that forms.

(iii) Find the average time a patient spends in the system.

(iv) What is the probability that there will be five or more patients waiting for the service?

(v) Determine the fraction of the time that there are no patients.

(vi) Find the average service time needs to be decreased to keep the average time in the system less than 25 minutes.

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Question 04

There are two clerks in a university to receive dues from the students. One clerk handles with 1^{st} and 2^{nd} year students and the other clerk handles with 3^{rd} and 4^{th} year students. It has been found that the service time distributions for students are exponential with mean service time 5 minutes per student. First and 2^{nd} year students are found to arrive in a Poisson distribution throughout the day with mean arrival rate 8 per hour. Third and 4^{th} year students also arrive in a Poisson distribution with mean arrival rate 6 per hour.

(a) What would be the effect on the average waiting time for students if each clerk could handle any student who comes from any year?

(b) What would be the effect if this could only be accomplished by increasing the service time 6 minutes?

Question 05

(a) Define the term "***inventory***".

(b) Write down the advantages and disadvantages of having inventories.

(c) Formulate the Economic Order Quantity (EOQ) model in which demand is not uniform and production rate is infinite.

Let $t_1, t_2, \ldots, t_n$ denote the times of successive production runs, such that

$$t_1 + t_2 + \ldots\ldots + t_n = 1 \text{ year}$$

(d) A company uses annually 24000 units of a raw material which costs Rs. 1.25 per unit. Placing each order costs Rs. 22.50, and the carrying cost is 5.4% of the average inventory. Find the Economic Order Quantity and the total inventory cost.

Question 06

(a) Briefly explain the following terms used in Inventory Management:

(i) Carrying cost

(ii) Shortage cost

(iii) Ordering cost

(b) Derive Economic Order Quantity model for deterministic demand when replenishment rate is infinite and shortages are permitted.

(c) A particular item has a demand of 9000 units per year. The cost of one procurement is Rs.100 and the holding cost per unit is Rs. 2.40 per year. The replacement is instantaneous and the cost of shortage is Rs. 5 per unit per year. Determine

(i) the lot size,

(ii) the number of orders per year,

(iii)the time between orders and

(iv)the total cost per year if the cost of one unit is Rs.1.

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Formulas (in the usual notation)

(M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) \qquad P(\text{queue size} \geq \text{n}) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu - \lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad E(v) = \frac{1}{\mu - \lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu - \lambda)}$$

(M/M/1): (N/FIFO) Queueing System

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1 - N\rho^{N-1} + (N-1)\rho^N\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(n) = \frac{\rho\left[1 - (N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})} \qquad E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1 - P_N)$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ; 1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ; n > C \end{cases} \qquad E(m) = \frac{\lambda\mu\left(\dfrac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu - \lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu - \lambda}\right]^{-1} \qquad E(w) = \frac{1}{\lambda}E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; 0 \le n \le C \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; C < n \le N \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=C}^{\infty} \frac{1}{C^{n-C}C!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C}\right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = {\left[E(n)\right]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; \; 0 \le n < R \\ \binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; \; R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right] \qquad E(v) = \frac{E(n)}{\lambda\left[K - E(n)\right]}$$

$$E(m) = \sum_{n=R}^{K}(n-R)P_n \qquad E(w) = \frac{E(m)}{\lambda\left[K - E(n)\right]}$$

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