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The Open University of Sri Lanka
B.Sc. Degree Programme, Level – 04
Final Examination – 2019/2020
PYU2165 – Mathematical Methods for Physics
Duration: 2 hours

Date: 23rd December 2019 **Time: 2.00 p.m. to 4.00 p.m.**

Answer any four (4) questions

Non-programmable calculators are allowed.

1.

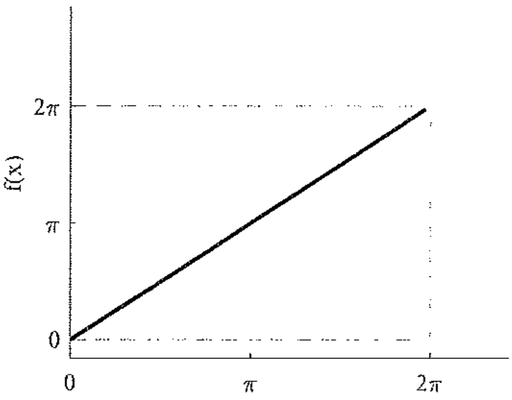
a. Calculate the Fourier series for the function $f(x) = x\,; 0 \le x \le 2\pi$. (Figure 1)

b. Integrate following expressions

i. $\int 3(6y^2 - 1)e^{2y^3 - y}.dy$
$Hint: u = 2y^3 - y$

ii. $\int xe^{6x}.dx$

iii. $\int_{-\pi}^{\pi} f(x).dx$ where
$$f(x) = \begin{cases} \sin(x) & x < 0 \\ -2\cos(x) & x \ge 0 \end{cases}$$



2.

a. Calculate the Eigen Values of A.

$$A = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$$

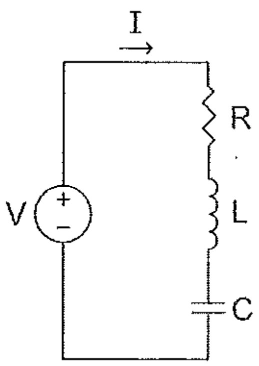
You may use a scientific calculator to solve cubic equations.

b. Following is an RLC circuit.

Impedance of each component is given below.

$$Z_R = R$$
$$Z_L = i\omega L$$
$$Z_C = \frac{1}{i\omega C}$$

R, L and C are constants. ω is the frequency.



Net Impedance of the circuit is given by $Z_{Net} = Z_R + Z_L + Z_C$

i. Obtain an expression for Z_{net} as a function of R, L, C *and* ω.

ii. At the resonance frequency, imaginary component of Z_{Net} will be zero, Calculate ω at resonance.

iii. Calculate $|Z_{Net}|$ at resonance.

3.

a. Ray transfer matrices allow calculation of the behavior of light beams. Let

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{Rn_2} & \frac{n_1}{n_2} \end{bmatrix}, C = \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}, D = \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}$$

i. If $n_1 = 1.5$, $n_2 = 1.0$, $R = 0.2$, $h_1 = 0.3$, $\theta_1 = 0.8$, rewrite matrices A_1, A_2 and C using numerical values.

ii. If $D = A_1 A_2 C$ calculate h_2, θ_2

b. Show that

$$det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -det \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$$

c. A water patch in the rainforest is invaded by an invasive water plant. Total area of the water patch is 120 m^2. A botanist one day visits the water patch and discovers 15 m^2 of the pond is covered by the invasive plants. He visits the site a week later and discovers it had spread to 30 m^2.

i. By assuming the spread of the invasive plant is arithmetic, calculate how long it will take to completely cover the pond.

ii. By assuming the spread of the invasive plant is geometric, calculate how long it will take to completely cover the pond.

iii. Is it possible to calculate the date at which the pond was first infected for each of the above cases? Explain.

4.

a. A researcher measures the temperature (T) above a certain land mass and found that it can be described by the following equation.

$$T(x,y,z) = 2x + 3y + \frac{xy}{2} - z^2 + 15$$

i. What is the temperature at the point (1,1,2)?

ii. What is the temperature at the point (2,2,3) ?

b.

A bird at the location Q (x,y,z) wish to fly in the direction of the highest temperature drop. Obtain an expression for this direction.

c.

Changes of temperature causes pressure changes which moves air molecules. Velocity of these molecules are described by the following equation.

$$\vec{V} = 2xy\hat{\imath} + 3xy\hat{\jmath} - x^2y^2\hat{k}$$

Obtain an expression for the net inward/outward flux of air molecules from a small volume around the point (x,y,z).

d.

Obtain an expression for the rotation of the air molecules around the point (x,y,z).

5.

A rare disease occurs in 1 in 2 million people in a population. A medical diagnosing test is developed to detect people with the illness. If a person has the disease, test always turns positive (i.e. there are no false negatives) . If a person doesn't have the disease, there is $\frac{1}{10^5}$ probability that the test might mistakenly turn positive (false positive).

a. In a country with 50 million people, estimate how many people having the disease. Similarly, estimate how many people not having the disease.

b. Government decides to screen everyone using the above test. How many people would get a "positive" result?

c. From the people who get a positive for the test, how many actually have the disease?

d. A person gets a positive in the test, what is the probability he actually has the disease?

e. If that person did the test again and test still become true, what is the probability he actually has the disease?

6.

a. Work done by an ideal gas during a reversible process is described by the following equation. 'w' is the total work done by the gas.'v' is the volume of the gas.

$$dw = P.dv$$

i. For an isothermal process, $P = \frac{NRT}{V}$ where N,R,T are constants. Show the total work by the change of volume from V_a *to* V_b is given by

$$w = nRTln\left(\frac{V_b}{V_a}\right)$$

ii. For an adiabatic process, $PV^{\gamma} = k$ where k is a constant. Show the total work done by the change of volume from V_a *to* V_b is given by

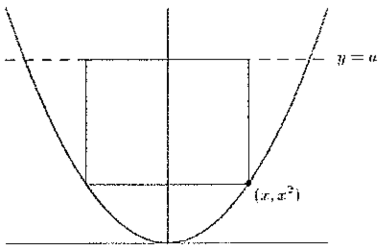
$$w = \frac{k\left(V_B^{(1-\gamma)} - V_A^{(1-\gamma)}\right)}{(1-\gamma)}$$

b. A student wish to find the largest rectangle that fits inside the graph of the parabola y = x^2 , below the line y = a (a is an unspecified constant), with top side of the rectangle on the line y = a.

i. Obtain expressions for the height and the width of the specified rectangle.

ii. Find the area of the rectangle as a function of x.

iii. Hence find the width and height of the largest rectangle which match the above criteria.



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