00145

The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Applied Mathematics – Level 04

ADU4303/ADE4303 – Applied Linear Algebra and Differential Equations

No Book Test (NBT) – 2019/2020

DURATION: ONE HOUR.

Date: 16 August 2020 **Time: 01.00 pm –02.00 pm**

ANSWER ALL QUESTIONS.

1. Find the general solution of the system of simultaneous differential equations

(i) $$\frac{dx_1}{dt} = 6x_1 - 3x_2 + e^{5t}$$
$$\frac{dx_2}{dt} = 2x_1 + x_2 + 4$$

(ii) $$\ddot{y}_1 = 3y_1 + 2(y_2 - y_1)$$
$$\ddot{y}_2 = -2(y_2 - y_1)$$

(iii) Use the change of variable, $x = \sin t\ (t > 0)$ to find the general solution of the equation

$$(1-x^2)\frac{d^2y}{dx^2} + \left((1-x^2)^{\frac{1}{2}} - x\right)\frac{dy}{dx} - 12y = 0, \quad (x > 0).$$

2. (i) Find the general solution of the following partial differential equation by using the integrating factor method.

$$\frac{\partial u}{\partial x} + \frac{1}{x^2(1+y)}u = -2(1-y)\exp\left(\frac{1}{x(1+y)}\right), \ (x \neq 0, y \neq -1).$$

(ii) (a) Suppose that u is a function of two variables x and t, satisfying the partial differential equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$, where c is a non-zero constant.

By transforming to new variables ζ and ϕ, where $\zeta = x - ct$ and $\phi = x + ct$,

show that the equation can be simplified to $\frac{\partial^2 u}{\partial \zeta \partial \phi} = 0$.

(b) Hence obtain the solution of the above equation.